

Integral-1

Integral – the “area” between the function and the x -axis.

“Area” above the x -axis is positive.

“Area” below the x -axis is negative.

Integration is the inverse of differentiation.

$$\text{So, } f' \left(\int f(x) dx \right) = f(x)$$

$$\text{And } \int f'(x) dx = f(x) \quad (\text{kind of})$$

$$f(x) = x^2$$

$$g(x) = x^2 + 4$$

$$h(x) = x^2 - 4$$

$$f'(x) = 2x$$

$$g'(x) = 2x$$

$$h'(x) = 2x$$

Note that **many** functions have the **same** derivative. This means that conversely, the integral of **one** function can be **many** functions!

$$\int f(x) dx = F(x) + c \quad (\text{indefinite integral})$$

$$\int_a^b f(x) dx = F(b) - F(a) \quad (\text{definite integral})$$

DESMOS Example:

$$\int_{-5}^5 x^2 dx = \frac{250}{3} \approx 83.3$$

$$\int_{-5}^5 4 dx = 40$$

$$\int_{-5}^5 (x^2 + 4) dx = \frac{370}{3} \approx 123.3$$

Note: this implies that $\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$

Integral-1

Determine the indefinite integral for each integral expression.

1. $\int 12x \, dx$

2. $\int x^2 \, dx$

3. $\int (x^2 + 3x + 2) \, dx$

4. $\int (x^2 + 1) \, dx$

5. $\int 8x^{-4} \, dx$

6. $\int (x + 1) \, dx$

7. $\int (9 - 5x) \, dx$

8. $\int (x^5 + 3x + 2) \, dx$

9. $\int 18x^{-4} \, dx$

10. $\int x^{-\frac{9}{5}} \, dx$

Answer Key

1. $6x^2 + c$

2. $\frac{1}{3}x^3 + c$

3. $\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x + c$

4. $\frac{1}{3}x^3 + x + c$

5. $-\frac{8}{3}x^{-3} + c$

6. $\frac{1}{2}x^2 + x + c$

7. $-\frac{5}{2}x^2 + 9x + c$

8. $\frac{1}{6}x^6 + \frac{3}{2}x^2 + 2x + c$

9. $-6x^{-3} + c$

10. $-\frac{5}{4}x^{-\frac{4}{5}} + c$